

# Holographic p-wave superconductors from Gauss-Bonnet gravity

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We study the holographic p-wave superconductors in a five-dimensional Gauss-Bonnet gravity with an  $SU(2)$  Yang-Mills gauge field. In the probe approximation, we find that when the Gauss-Bonnet coefficient grows, the condensation of the vector field becomes harder, both the perpendicular and parallel components, with respect to the direction of the condensation, of the anisotropic conductivity decrease. We also study the mass of the quasi-particle excitations, the gap frequency and the DC conductivities of the p-wave superconductor. All of them depend on the Gauss-Bonnet coefficient. In addition we observe a strange behavior for the condensation and the relation between the gap frequency and the mass of quasi-particles when the Gauss-Bonnet coefficient is larger than  $9/100$ , which is the upper bound for the Gauss-Bonnet coefficient from the causality of the dual field theory.

## I. INTRODUCTION

The AdS/CFT correspondence [1–4] provides a theoretical method to understand strongly coupled field theories. It has been applied to calculate transport coefficients, such as shear viscosity, of strongly coupled systems and some universal properties of dual strongly coupled field theories have been found in the hydrodynamical limit [5–8]. Recently, it has been proposed that the AdS/CFT correspondence also can be used to describe superconductor phase transition [9, 10]. Since the high  $T_c$  superconductors are shown to be in the strong coupling regime, one expects that the holographic method could give some insights into the pairing mechanism in the high  $T_c$  superconductors.

There have been lots of work studying various holographic superconductors [11–35], in which some effects such as scalar field mass, external magnetic field, and back reaction *etc.* have been discussed. Among those works, some universality is discovered. For instance, the ratio of gap frequency over critical temperature  $\omega_g/T_c$  is found to be always near the value 8 [12], which is more than twice the weakly coupled BCS theory value 3.5. The reason for this bigger value might be that the holographic superconductor is strongly coupled.

In the holographic study of shear viscosity, the universal bound on the ratio of the shear

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viscosity over entropy density  $\eta/s \geq 1/4\pi$  is found to be violated in theories dual to gravity systems with some higher curvature corrections [36–43]. This promotes the study of holographic superconductors under higher curvature corrections [44–49]. Another motivation to consider the higher curvature effect on the holographic superconductors is due to the Mermin-Wagner theorem or Coleman theorem, which states that in quantum field theory, continuous symmetries cannot be spontaneously broken at finite temperature in systems with sufficiently short-range interactions in spatial dimensions  $d \leq 2$ . However, one indeed observes the superconducting phase transition in the gravity dual of four-dimensional AdS black hole backgrounds [9, 10]. This might be caused by the suppression of the large fluctuations in the large  $N$  limit, which is supposed to be one of conditions for the validity of the AdS/CFT correspondence. In [44, 45] the effect of the Gauss-Bonnet term on the holographic s-wave superconductors has been investigated. It is found that a larger Gauss-Bonnet term makes condensation harder, the universality of  $\omega_g/T_c \approx 8$  is violated and the ratio of gap frequency over critical temperature depends on the Gauss-Bonnet coefficient.

Besides the intensively studied holographic s-wave superconductors, there also exist holographic p-wave superconductor models [50]. In the p-wave case, there is a special direction which breaks the rotational symmetry. In Ref. [50], the authors studied the p-wave superconductors and observed an anisotropic conductivity. Their results fit well with the Drude model in the low frequency limit. Further the authors of [17] studied the back reaction effect of matter field on the p-wave superconductor and found that when the ratio of the five-dimensional gravitational constant to the Yang-Mills coupling is beyond a critical value ( $\approx 0.365$ ), the phase transition becomes first order.

In this paper, we are interested in the effect of the Gauss-Bonnet term on the 4-dimensional p-wave superconductors, which are dual to 5-dimensional Gauss-Bonnet-AdS black holes in the bulk. In the probe approximation, the bulk spacetime is a 5-dimensional Gauss-Bonnet-AdS black hole with a Ricci flat horizon [52]. We find that the condensation increases as the Gauss-Bonnet coefficient grows, which means a positive Gauss-Bonnet term makes the condensation harder as the case of s-wave superconductors. There are two different kinds of conductivity due to the anisotropic condensation in p-wave superconductors. The conductivity perpendicular to the direction of the condensation behaves like a s-wave one. On the other hand, the conductivity parallel to the direction of the condensation behaves much different. In the low frequency regime, this conductivity can be well explained by Drude model as in Ref. [50]. In addition, we see that the Gauss-Bonnet term will not change the order of the phase transition. Namely the phase transition is still second order and some critical exponents still take their mean-field theory values.

This paper is organized as follows. In Sec. II, we give out the basic setup and study the superconductor phase transition in the probe limit with various values of the Gauss-Bonnet coefficient. In Sec. III, we calculate the anisotropic frequency dependent conductivity. We conclude our paper in Sec. IV.

## II. HOLOGRAPHIC P-WAVE SUPERCONDUCTOR

We consider the bulk theory of the Einstein-Gauss-Bonnet gravity with an SU(2) Yang-Mills field in a 5-dimensional space-time. The action is

$$S = \int d^5x \sqrt{-g} \left[ \frac{1}{2\kappa_5^2} \left( R + \frac{12}{L^2} + \frac{\alpha}{2} (R^2 - 4R^{\mu\nu} R_{\mu\nu} + R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}) \right) - \frac{1}{4\hat{g}^2} (F_{\mu\nu}^a F^{a\mu\nu}) \right], \quad (1)$$

where  $\kappa_5$  is the five dimensional gravitational constant,  $\hat{g}$  is the Yang-Mills coupling constant and  $L$  the radius of the AdS spacetime.  $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \epsilon^{abc} A_\mu^b A_\nu^c$  is the Yang-Mills field strength,  $\epsilon^{abc}$  is the totally antisymmetric tensor with  $\epsilon^{123} = +1$ . The quadratic curvature term is the Gauss-Bonnet term with  $\alpha$  the Gauss-Bonnet coefficient.

We are interested in the asymptotic AdS solution to this gravity system. In general, we should solve the Yang-Mills equations as well as gravitational field equations to search for a required solution. To solve this problem with the Gauss-Bonnet term will be difficult. However, we can get some qualitative features in the so-called probe limit, where the back reaction of matter fields (Yang-Mills field) on the metric can be neglected. This approximation can be justified. Indeed, we can see from the action that in the limit  $\kappa_5^2/\hat{g}^2 \ll 1$ , the back reaction of the Yang-Mills field on the metric can be neglected safely.

In the probe limit, the background metric is a 5-dimensional Gauss-Bonnet-AdS black hole with a Ricci flat horizon [52]. The metric is described by

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + \frac{r^2}{L^2}(dx^2 + dy^2 + dz^2), \quad (2)$$

$$f(r) = \frac{r^2}{2\alpha} \left( 1 - \sqrt{1 - \frac{4\alpha}{L^2} \left( 1 - \frac{mL^2}{r^4} \right)} \right),$$

where  $m$  is the mass of the black hole. The horizon is located at  $r = r_h = \sqrt[4]{mL^2}$ , and the temperature of the black hole is

$$T = \frac{r_h}{\pi L^2}. \quad (3)$$

Here we should notice that in the asymptotic region with  $r \rightarrow \infty$ ,

$$f(r) \sim \frac{r^2}{2\alpha} \left[ 1 - \sqrt{1 - \frac{4\alpha}{L^2}} \right]. \quad (4)$$

One can define an effective radius  $L_{\text{eff}}$  of the AdS spacetime by

$$L_{\text{eff}}^2 \equiv \frac{2\alpha}{1 - \sqrt{1 - \frac{4\alpha}{L^2}}}. \quad (5)$$

We can see from this equation that one has to have  $\alpha \leq L^2/4$  in order to have a well-defined vacuum for the gravity theory. The upper bound  $\alpha = L^2/4$  is called Chern-Simons limit. In the AdS/CFT correspondence, this asymptotic AdS spacetime is dual to a conformal field theory living on the boundary  $r \rightarrow \infty$ . The temperature of the black hole is just the one of the dual field theory. If we further consider the causality constraint from the boundary

CFT, there is an additional constraint on the Gauss-Bonnet coefficient with  $-7L^2/36 \leq \alpha \leq 9L^2/100$  in five dimensions [36, 37, 53–57].

The probe SU(2) Yang-Mills field  $A_\mu^a$  is dual to some current operator  $J_\mu^a$  in the 4-dimensional boundary theory. In order to realize a holographic p-wave superconductor, following Ref. [50, 51] we adopt the ansatz

$$A = \phi(r)\tau^3 dt + \psi(r)\tau^1 dx \quad (6)$$

for the Yang-Mills gauge field. Here  $\tau^i$ 's are the three SU(2) generators with commutation relation  $[\tau^i, \tau^j] = \epsilon^{ijk}\tau^k$ . In this ansatz one can regard the U(1) subalgebra generated by  $\tau^3$  as the gauge group of electromagnetism, and then the condensation of  $\psi(r)$  will break the U(1) symmetry and lead to the superconductor phase transition. Because  $\psi(r)$  is dual to the  $J_x^1$  operator on the boundary, choosing x-axis as a special direction, the condensation of  $\psi(r)$  breaks the rotational symmetry and leads to a phase transition, which can be interpreted as a p-wave superconducting phase transition on the boundary.

The Yang-Mills equations with the above ansatz (6) are

$$\begin{cases} \phi'' + \frac{3}{r}\phi' - \frac{L^2\psi^2}{r^2 f}\phi = 0, \\ \psi'' + \left(\frac{1}{r} + \frac{f'}{f}\right)\psi' + \frac{\phi^2}{f^2}\psi = 0. \end{cases} \quad (7)$$

We'll solve the Yang-Mills equations on the black hole background (2) and study the solutions with non-zero  $\psi(r)$  which is related to the p-wave superconducting phase.

In order to solve the equations (7), we need the boundary conditions for the fields  $\psi(r)$  and  $\phi(r)$ . On the black hole horizon, it is required that  $\phi(r_h) = 0$  for the U(1) gauge field to have a finite norm, and  $\psi(r_h)$  should be finite. Therefore, the boundary conditions of  $\psi$  and  $\phi$  on the horizon are:

$$\psi = \psi_H^{(0)} + \psi_H^{(2)}\left(1 - \frac{r_h}{r}\right)^2 + \dots, \quad (8)$$

$$\phi = \phi_H^{(1)}\left(1 - \frac{r_h}{r}\right) + \dots. \quad (9)$$

On the boundary of the bulk, we have

$$\phi(r) \rightarrow \mu + \rho/r^2 \quad (10)$$

$$\psi(r) \rightarrow \psi^{(0)} + \psi^{(2)}/r^2. \quad (11)$$

$\mu$  and  $\rho$  are dual to the chemical potential and charge density of the boundary CFT,  $\psi^{(0)}$  and  $\psi^{(2)}$  are dual to the source and expectation value of the boundary operator  $J_x^1$  respectively. We always set the source  $\psi^{(0)}$  to zero, as we want to have a normalizable solution.

The trivial solution to the above Yang-Mills equations is a charged black hole solution with  $\psi(r) = 0$ , which is just the non-superconducting phase in the boundary theory. We will try to find non-trivial solutions, describing the p-wave superconducting phase, with  $\psi(r) \neq 0$ . We use a shooting method in which we solve the Yang-Mills equations numerically from the horizon to boundary, and pick the suitable one obeying the boundary conditions at  $r \rightarrow \infty$ .

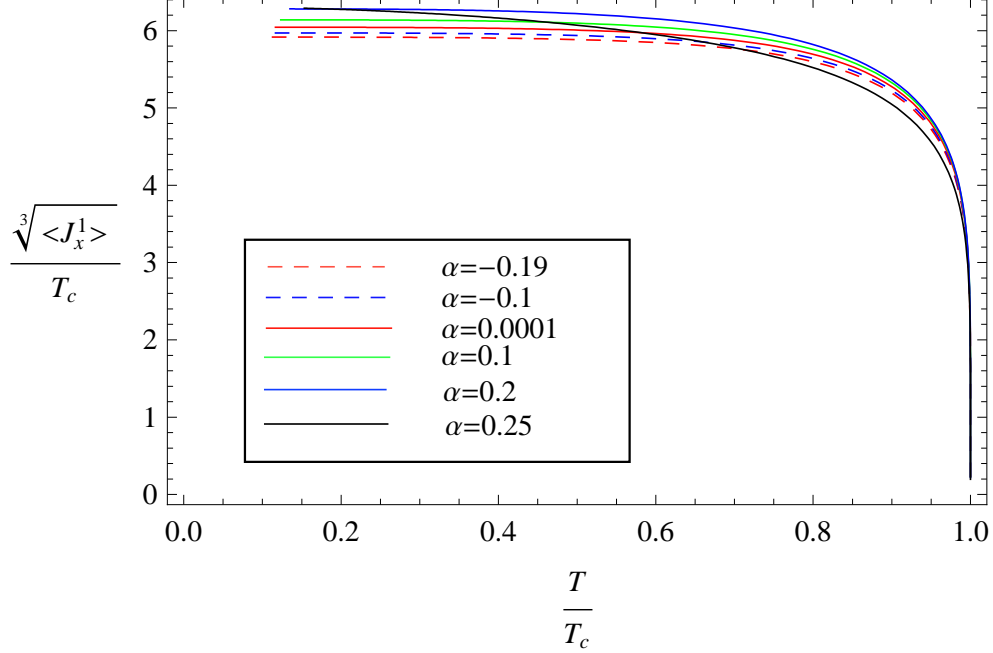


FIG. 1: The condensation of  $J_x^1$  as a function of temperature. The six lines correspond to different Gauss-Bonnet coefficient, respectively: The dashed red line to  $\alpha = -0.19$ , the dashed blue line to  $\alpha = -0.1$ , the red line to  $\alpha = 0.0001$ , the green line to  $\alpha = 0.1$ , the blue line to  $\alpha = 0.2$ , and the black line to  $\alpha = 0.25$  which saturates the Chern-Simons limit.

In the numerical calculation, we set  $L = 1$  and define a new variable  $z = r_h/r$ . Rescaling  $r$ ,  $\phi(r)$  and  $\psi(r)$ , one then can simply set  $r_h = 1$ .

In Figure 1, we plot the condensation of  $J_x^1$  as a function of temperature with various values of the Gauss-Bonnet coefficient  $\alpha$ . Note that the boundary operator  $J_x^1$  is a component of a vector operator. Unlike the scalar field in the holographic s-wave superconductor, one can find in the AdS/CFT dictionary [4] that the conformal dimension of  $J_x^1$  is  $\lambda = 3$ , the same as the charge density  $\rho$ . In the figure, we therefore plot the data of  $J_x^1$  as  $\sqrt[3]{J_x^1}/T_c$ , as this is the right dimensionless quantity. We can see from the figure that the condensation value increases with the increase of the Gauss-Bonnet coefficient. This is the same as the s-wave case [44, 45]. Note that the curve for the case  $\alpha = 0.25$  intersects with others. We have some to say later on this behavior.

In mean-field theory, the condensation of operator is proportional to  $\sqrt{(1 - T/T_c)}$  when  $T \rightarrow T_c$ . We can see from the figure 1 that the p-wave condensations have a similar behavior like that in mean-field theory when  $T \rightarrow T_c$ , we fit the data as follows:

$$\begin{aligned}
 \alpha = -0.19, \quad & \langle J_x^1 \rangle = 476.2698 T_c^3 (1 - T/T_c)^{1/2}, \\
 \alpha = -0.1, \quad & \langle J_x^1 \rangle = 487.0347 T_c^3 (1 - T/T_c)^{1/2}, \\
 \alpha = 0.0001, \quad & \langle J_x^1 \rangle = 499.0358 T_c^3 (1 - T/T_c)^{1/2}, \\
 \alpha = 0.1, \quad & \langle J_x^1 \rangle = 509.7952 T_c^3 (1 - T/T_c)^{1/2}, \\
 \alpha = 0.2, \quad & \langle J_x^1 \rangle = 519.5827 T_c^3 (1 - T/T_c)^{1/2}, \\
 \alpha = 0.25, \quad & \langle J_x^1 \rangle = 422.3983 T_c^3 (1 - T/T_c)^{1/2}.
 \end{aligned} \tag{12}$$

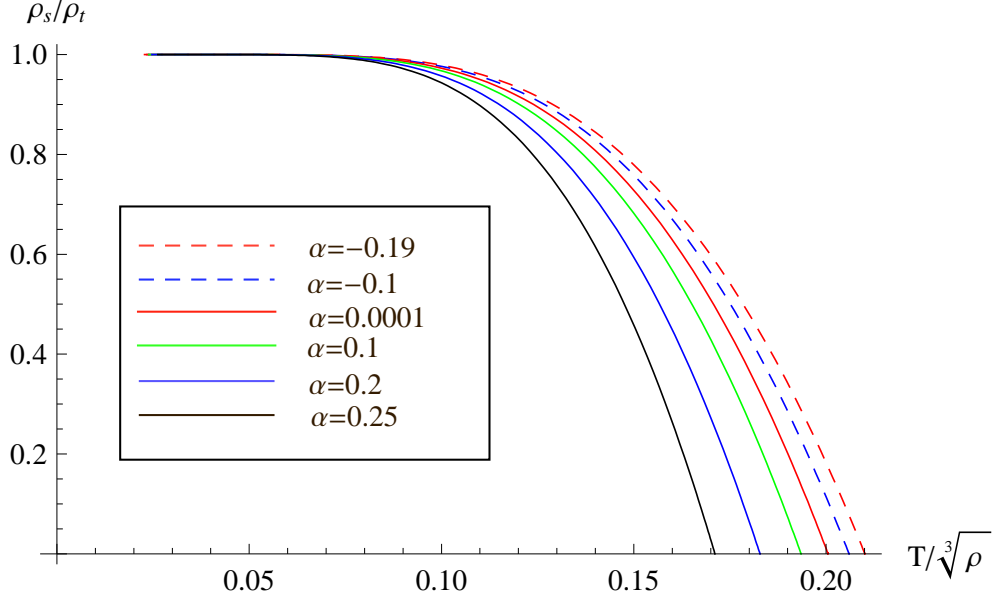


FIG. 2: The ratio of the superconducting charge density to the total charge density versus temperature. The intersecting points of the curves with the horizontal axis represent the critical temperature  $T_c$  when the superconducting phase occurs.  $T_c$  decreases when  $\alpha$  increases.

This indicates that the Gauss-Bonnet term does not change the critical exponent of the condensation.

In the holographic p-wave superconductor, the normal charge density  $\rho_n$  is  $\rho_n = \phi_H^{(1)}$ , and the total charge density is  $\rho_t = 2\rho$ , the factor 2 arises due to the scaling behavior of  $\phi$  on the infinite boundary. The superconducting charge density is defined as  $\rho_s = \rho_t - \rho_n$ . We plot the ratio  $\rho_s/\rho_t$  in Figure 2. The points where the curves intersect with the horizontal axis represent the critical temperatures for different  $\alpha$  when the superconducting phase occurs. We can see from the figure that the critical temperature decreases with the increase of the Gauss-Bonnet coefficient. In fact,

$$\begin{aligned}
 T_c &= 0.2101\sqrt[3]{\rho} & \text{when } \alpha &= -0.19, \\
 T_c &= 0.2060\sqrt[3]{\rho} & \text{when } \alpha &= -0.1, \\
 T_c &= 0.2005\sqrt[3]{\rho} & \text{when } \alpha &= 0.0001, \\
 T_c &= 0.1935\sqrt[3]{\rho} & \text{when } \alpha &= 0.1, \\
 T_c &= 0.1828\sqrt[3]{\rho} & \text{when } \alpha &= 0.2, \\
 T_c &= 0.1711\sqrt[3]{\rho} & \text{when } \alpha &= 0.25.
 \end{aligned} \tag{13}$$

So we conclude that a larger Gauss-Bonnet term makes the condensation harder to form. This result is qualitatively the same as the Gauss-Bonnet effect on the holographic s-wave superconductors [44–46].

In the next section, we will calculate the frequency dependent conductivity to see the influence of the Gauss-Bonnet term more clearly.

### III. ELECTRIC CONDUCTIVITY

In order to see the electric conductivity of the system, we can add an electromagnetic perturbation into the system. For the Yang-Mills case, the perturbation of the electric field will mix other components in the linearized equation [50]. The perturbation is  $A \rightarrow A + \delta A$ , where

$$\delta A = e^{-i\omega t}[(A_t^1(r)\tau^1 + A_t^2(r)\tau^2)dt + A_x^3(r)\tau^3dx + A_y^3(r)\tau^3dy]. \quad (14)$$

Note that there still exists a  $SO(2)$  symmetry in the plane  $y - z$  in the system. The electric conductivity  $\sigma_{zz}$  is completely the same as  $\sigma_{yy}$ . Therefore we have neglected the perturbation along the direction  $z$  for simplicity. In the following subsections, we separately calculate the components  $\sigma_{yy}$  and  $\sigma_{xx}$  to see the difference between them, and study the effect of the Gauss-Bonnet term on the electric conductivities.

#### A. $\sigma_{yy}$

The linearized equation of motion for  $A_y^3$  decouples from other components of the Yang-Mills field. The equation of motion is

$$\left(\frac{\omega^2}{f^2} - \frac{L^2\psi^2}{r^2f}\right)A_y^3 + \left(\frac{1}{r} + \frac{f'}{f}\right)A_y^{3'} + A_y^{3''} = 0. \quad (15)$$

The equation (15) is very similar to corresponding equation in the holographic s-wave superconductors [9, 10, 50]. Therefore, the calculation of  $\sigma_{yy}$  is the same as that in the s-wave case. But we still show its details here in order to make a contrast to  $\sigma_{xx}$ .

To calculate the conductivity, we impose the in-falling wave condition to  $A_y^3$  on the horizon. Then the current Green's function with zero spatial momentum  $G^R(\omega, \vec{0})$  can be evaluated using AdS/CFT correspondence as [27, 58]

$$G^R(\omega, \vec{0}) = -\lim_{r \rightarrow \infty} r f A_y^3 A_y^{3'}. \quad (16)$$

Here,  $A_y^3(r)$  is normalized to be  $A_y^3(r \rightarrow \infty) = 1$ . Near the boundary of the AdS bulk, the expansion of  $A_y^3$  is

$$A_y^3 = A_y^{3(0)} + \frac{A_y^{3(2)}}{r^2} + \frac{A_y^{3(0)}\omega^2 L_{\text{eff}}^2 \log \Lambda r}{2r^2}. \quad (17)$$

where,  $L_{\text{eff}}^2$  is the one in the formula (5), and  $\Lambda$  is an arbitrary constant. Thus the retarded Green's function is

$$G^R(\omega, \vec{0}) = 2 \frac{A_y^{3(2)}}{A_y^{3(0)} L_{\text{eff}}^2} + \omega^2 L_{\text{eff}}^2 (\log \Lambda r - \frac{1}{2}). \quad (18)$$

The logarithmic divergence can be removed with a boundary counterterm in the gravity action [59]. This procedure is related to renormalization of the UV divergence in the boundary theory. After adding a counterterm to cancel the logarithmic divergence, we have the retarded Green's function

$$G^R(\omega, \vec{0}) = 2 \frac{A_y^{3(2)}}{A_y^{3(0)} L_{\text{eff}}^2} - \frac{1}{2} \omega^2 L_{\text{eff}}^2, \quad (19)$$

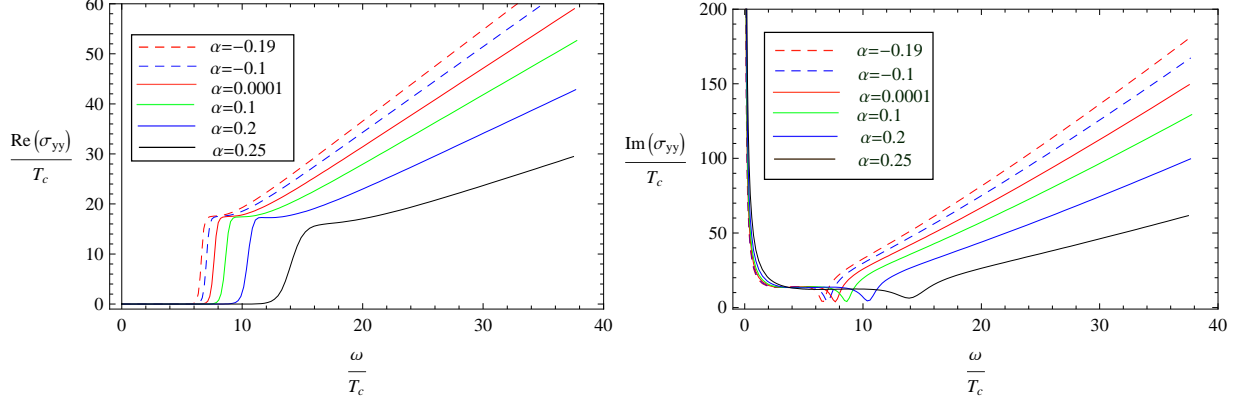


FIG. 3: The conductivity  $\sigma_{yy}$  as a function of frequency.

and the conductivity is

$$\sigma_{yy} = \frac{1}{i\omega} G^R(\omega, \vec{0}) = \frac{-2iA_y^{3(2)}}{A_y^{3(0)}\omega L_{\text{eff}}^2} + \frac{i}{2}\omega L_{\text{eff}}^2. \quad (20)$$

We numerically solve the equation of motion (15) with the boundary conditions mentioned above and obtain  $\sigma_{yy}$ . The results are plotted in Figure 3 with various values of the Gauss-Bonnet coefficient. We can clearly see from the figure that for a fixed frequency, both the real and imaginary parts of the conductivity  $\sigma_{yy}$  decrease as the increase of the Gauss-Bonnet coefficient. In addition, from the conductivity, one can conclude that the ratio of gap frequency over the critical temperature increases when the Gauss-Bonnet coefficient becomes large.

### B. $\sigma_{xx}$

The calculation of  $\sigma_{xx}$  is more complicated. The equation of motion for  $A_x^3$  mixes it with other two components  $A_t^1$  and  $A_t^2$  in (14). The equations of motion for the three components are

$$A_x^{3''} + \left(\frac{1}{r} + \frac{f'}{f}\right)A_x^{3'} + \frac{\omega^2 A_x^3}{f^2} - \frac{i\omega A_t^2 + A_t^1 \phi}{f^2} \psi = 0, \quad (21)$$

$$A_t^{1''} + \frac{3A_t^{1'}}{r} + \frac{L^2 A_x^3 \phi \psi}{r^2 f} = 0, \quad (22)$$

$$A_t^{2''} + \frac{3A_t^{2'}}{r} - \frac{L^2 A_t^2 \psi^2}{r^2 f} - \frac{iL^2 \omega A_x^3 \psi}{r^2 f} = 0. \quad (23)$$

Besides these three equations of motion, there are two additional constraints:

$$-\phi A_t^{1'} + i\omega A_t^{2'} - \frac{L^2 f \psi A_x^{3'}}{r^2} + A_t^1 \phi' + \frac{L^2 A_x^3 f \psi'}{r^2} = 0, \quad (24)$$

$$i\omega A_t^{1'} + \phi A_t^{2'} - A_t^2 \phi' = 0. \quad (25)$$



The constraint equations (24) and (25) are not independent of the equations of motion (21),(22) and (23). Actually, the derivatives of the two constraint equations follow algebraically from the equations of motion.

Again we focus on the solutions with the in-falling wave conditions on the horizon, which determine the retarded Green's function. With the in-falling wave condition, we get from Eqs. (21),(22) and (23) that near the horizon,

$$A_x^3 = (1 - \frac{r_h}{r})^{-i\omega L^2/(4r_h)} [1 + a_x^{3(1)}(1 - \frac{r_h}{r}) + a_x^{3(2)}(1 - \frac{r_h}{r})^2 + \dots], \quad (26)$$

$$A_t^1 = (1 - \frac{r_h}{r})^{-i\omega L^2/(4r_h)} [a_t^{1(2)}(1 - \frac{r_h}{r})^2 + a_t^{1(3)}(1 - \frac{r_h}{r})^3 + \dots], \quad (27)$$

$$A_t^2 = (1 - \frac{r_h}{r})^{-i\omega L^2/(4r_h)} [a_t^{2(1)}(1 - \frac{r_h}{r}) + a_t^{2(2)}(1 - \frac{r_h}{r})^2 + \dots]. \quad (28)$$

where  $a_\mu^{a(i)}$  are some constants. The boundary conditions on the boundary of the AdS bulk can also be read from the expansion of Eqs. (21), (22) and (23). They are:

$$A_x^3 = A_x^{3(0)} + \frac{A_x^{3(2)}}{r^2} + \frac{A_x^{3(0)}\omega^2 L_{\text{eff}}^4 \log(\Lambda r)}{2r^2} + \dots, \quad (29)$$

$$A_t^1 = A_t^{1(0)} + \frac{A_t^{1(2)}}{r^2} + \dots, \quad (30)$$

$$A_t^2 = A_t^{2(0)} + \frac{A_t^{2(2)}}{r^2} + \dots. \quad (31)$$

These coefficients in the expansions can be fixed using the equations of motion and the constraint equations.

Here the calculation of conductivity is more subtle than that of  $\sigma_{yy}$ . Because the definition of  $A_x^3$  depends on the choice of gauge, we cannot obtain the conductivity straightforwardly by using the solution of  $A_x^3$  only. What we needed is a physical combination of  $A_x^3$ ,  $A_t^1$  and  $A_t^2$ , in other words, a gauge invariant quantity. As argued in Ref. [50], this gauge invariant quantity should be

$$\hat{A}_x^3 = A_x^3 + \psi \frac{i\omega L^2 A_t^2 + \phi A_t^1}{\phi^2 - \omega^2 L^4}. \quad (32)$$

This gauge invariant quantity near the boundary is

$$\hat{A}_x^3 = A_x^{3(0)} + \frac{\hat{A}_x^{3(2)}}{r^2} + \frac{A_x^{3(0)}\omega^2 L_{\text{eff}}^4 \log(\Lambda r)}{2r^2}, \quad (33)$$

where

$$\hat{A}_x^{3(2)} \equiv A_x^{3(2)} + \psi^{(2)} \frac{i\omega L^2 A_t^{2(0)} + \mu A_t^{1(0)}}{\mu^2 - \omega^2 L^4}. \quad (34)$$

With  $\hat{A}_x^3$ , we can compute the conductivity  $\sigma_{xx}$  as

$$\sigma_{xx} = \frac{1}{i\omega} G^R(\omega, k=0) = -\frac{2i\hat{A}_x^{3(2)}}{A_x^{3(0)}\omega L_{\text{eff}}^2} + \frac{1}{2}i\omega L_{\text{eff}}^2. \quad (35)$$

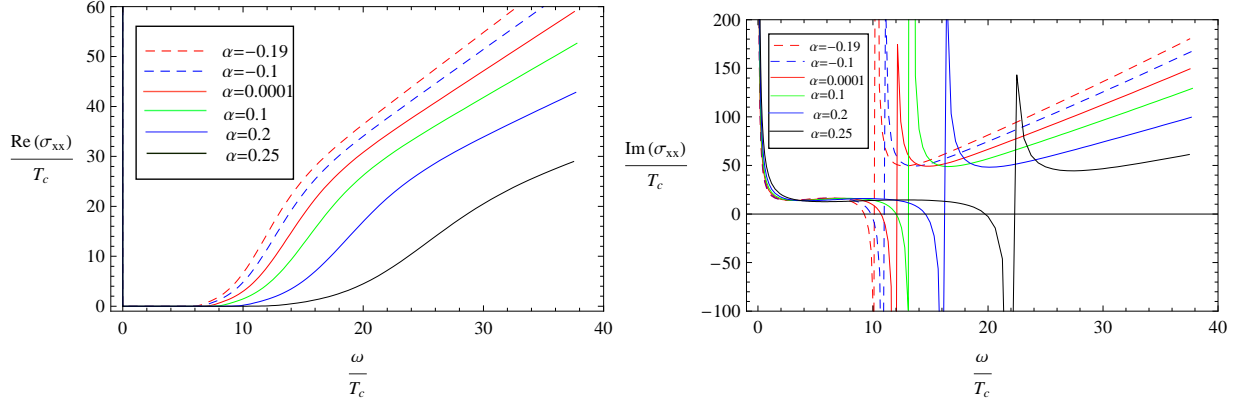


FIG. 4: The conductivity  $\sigma_{xx}$  as a function of frequency.

The results are plotted in Figure 4. Comparing with Figure 3, we can see that the conductivity  $\sigma_{xx}$  behaves quite different from  $\sigma_{yy}$ . The real part of the  $\sigma_{xx}$  grows much slowly than that of  $\sigma_{yy}$ . The anisotropic behavior of conductivity is just the feature of p-wave superconductors.

### C. More on $\sigma_{xx}$ and $\sigma_{yy}$

From Figure 3, we see that the real part of the conductivity of  $\sigma_{yy}$  rises quickly around some frequency. This behavior is very similar to the case of holographic s-wave superconductors [10–12]. In particular, the growing behavior of the conductivity for large frequency is due to the term  $\log(\Lambda r)$  in the expansion (29). This behavior of the conductivity is the main discrepancy between (3+1)-dimensional and (2+1)-dimensional s-wave superconductor [12]; in the latter case, the real part of the conductivity approaches to a constant when  $\omega \rightarrow \infty$ . We can define  $\omega_g$  as the gap frequency where the imaginary part of the conductivity  $\text{Im}\sigma_{yy}$  minimizes as in [12], which describes excitation of quasi-particles in pairs. The normal contribution to the DC conductivity of  $\sigma_{yy}$  is defined as  $n_n = \lim_{\omega \rightarrow 0} \text{Re}(\sigma)$  which is exponentially suppressed as  $n_n \sim e^{-\Delta/T}$ , where  $\Delta$  is the mass of excited quasi-particles. In the BCS theory,  $\omega_g = 2\Delta$ . However, in holographic superconductors, this relation no longer holds in general [12]. This might be caused by strong coupling between quasi-particles.

From the right panel of Figure 3, we read off

$$\begin{aligned} \omega_{g(\alpha=-0.19)} &\approx 6.6T_c, & \omega_{g(\alpha=-0.1)} &\approx 7.0T_c, \\ \omega_{g(\alpha=0.0001)} &\approx 7.7T_c, & \omega_{g(\alpha=0.1)} &\approx 8.6T_c, \\ \omega_{g(\alpha=0.2)} &\approx 10.5T_c, & \omega_{g(\alpha=0.25)} &\approx 14.0T_c. \end{aligned} \quad (36)$$

As in the case of holographic s-wave superconductors, the ratio of the gap frequency over the critical temperature  $\omega_g/T_c$  deviates from the universal value 8 and the ratio depends on the Gauss-Bonnet coefficient. We can see that  $\omega_g$  increases as  $\alpha$  grows. Because  $\omega_g$  can be interpreted as the energy to break a pair of fermions, the bigger  $\omega_g$  is, the harder the fermion pairs to form. Thus we can draw the conclusion again that a positive Gauss-Bonnet

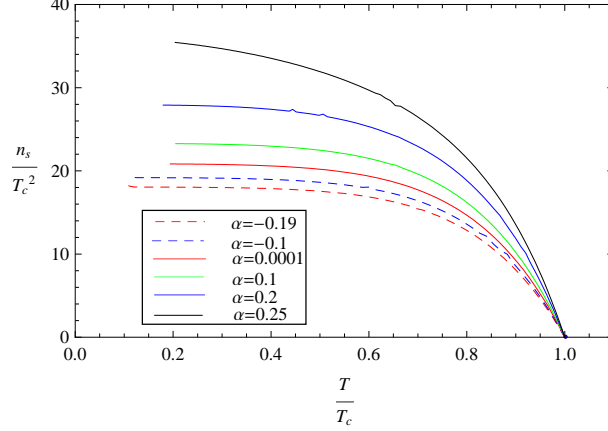


FIG. 5: The superfluid density of  $\sigma_{yy}$  for various  $\alpha$ .

term makes the condensation harder. Furthermore, because the boundary spacetime is four dimensional, the conductivity is of mass dimension one. Therefore, we can read off  $\Delta$  from the dimensionless quantity  $\text{Re}(\sigma)/T_c$ . They are

$$\begin{aligned} \Delta_{\alpha=-0.19} &\approx 4.62T_c, & \Delta_{\alpha=-0.1} &\approx 4.64T_c \\ \Delta_{\alpha=0.0001} &\approx 4.69T_c, & \Delta_{\alpha=0.1} &\approx 4.77T_c, \\ \Delta_{\alpha=0.2} &\approx 4.94T_c, & \Delta_{\alpha=0.25} &\approx 5.28T_c, \end{aligned} \quad (37)$$

for different Gauss-Bonnet coefficient. Comparing (36) with (37), we can clearly see that in this holographic model of p-wave superconductor,  $\omega_g \neq 2\Delta$ . In particular, we notice that one has  $\omega_g < 2\Delta$ . We can explain the difference  $2\Delta - \omega_g$  as the bound energy between a pair of quasi-particles. Two exceptions are the cases of  $\alpha = 0.2$  and  $\alpha = 0.25$ , for which  $\omega_g > 2\Delta$ . Let us recall that the condensation behavior for the case of  $\alpha = 0.25$  behaves strange (see Figure 1). Note that the causality condition for the dual field theory leads to a constraint on the Gauss-Bonnet coefficient  $-7/36 \leq \alpha \leq 9/100$ . Thus we can conclude that strange behavior for the case of  $\alpha = 0.25$  in fact demonstrates that the results for the cases  $\alpha = 0.20$ , and  $0.25$  are not trustable since the dual field theories are not well-defined.

Another important order parameter of s-wave superconductors is the superfluid density  $n_s$ . It can be related to the retarded Green's function as  $n_s = \text{Re}[G^R(\omega = k = 0)]$ . The behavior of the dimensionless quantity  $n_s/T_c^2$  is plotted in Figure 5 for various Gauss-Bonnet coefficient  $\alpha$ . Near the critical temperature,  $T \rightarrow T_c$ , the superfluid density is linearly proportional to  $(T_c - T)$  as follows:

$$\begin{aligned} n_{s(\alpha=-0.19)} &\approx 87.54T_c(T_c - T), & n_{s(\alpha=-0.1)} &\approx 93.40T_c(T_c - T) \\ n_{s(\alpha=0.0001)} &\approx 98.70T_c(T_c - T), & n_{s(\alpha=0.1)} &\approx 111.14T_c(T_c - T), \\ n_{s(\alpha=0.2)} &\approx 126.37T_c(T_c - T), & n_{s(\alpha=0.25)} &\approx 146.14T_c(T_c - T). \end{aligned} \quad (38)$$

This linear behavior is the same as in the four dimensional case [50]. In addition we can see that the factors of the linear relation increase as  $\alpha$  grows. Here it shows again that the Gauss-Bonnet term does not change the critical exponent associated with the superfluid density.

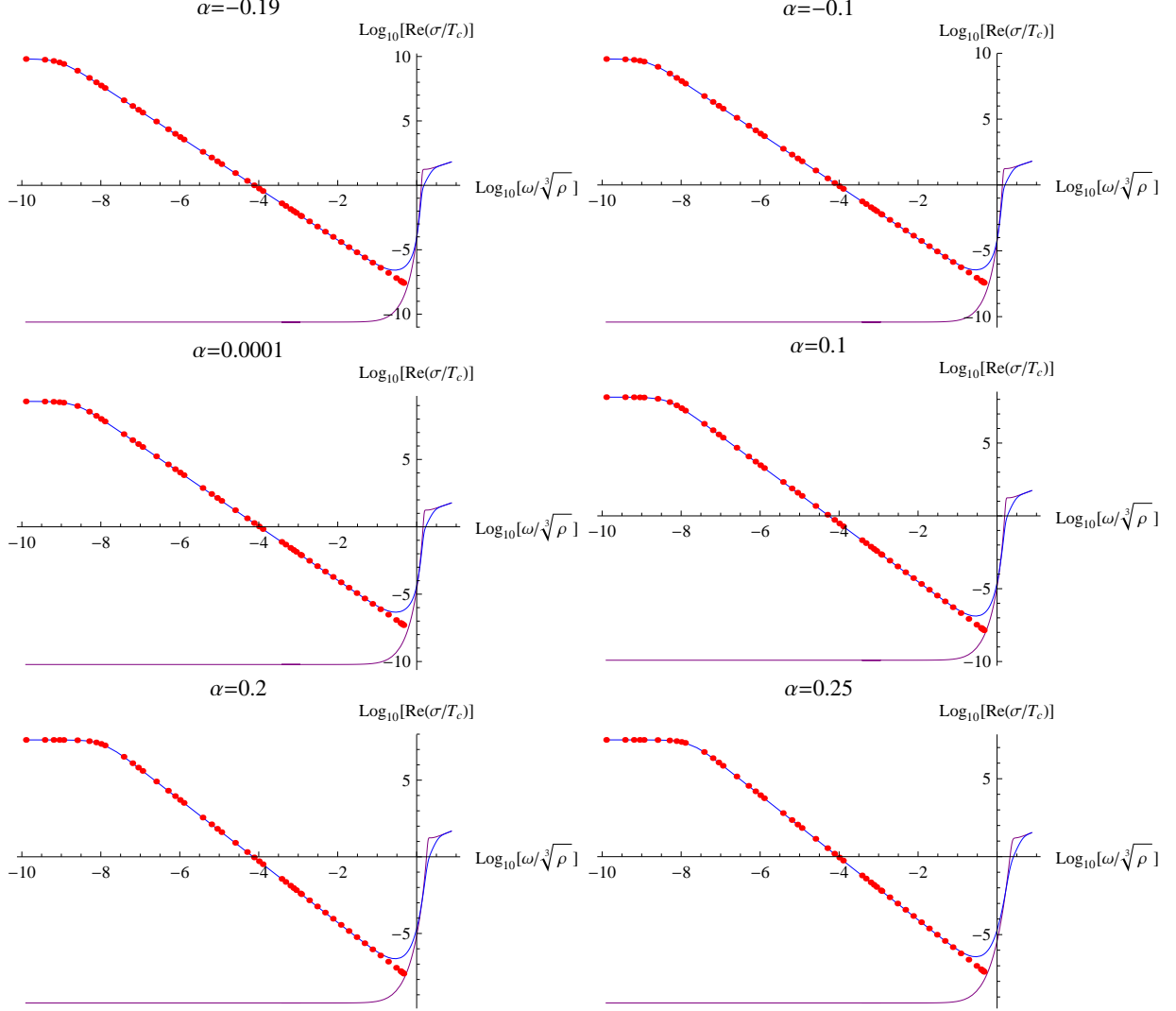


FIG. 6: Logarithmic real parts of conductivity versus the logarithmic frequencies for various  $\alpha$ . The blue line represents  $\sigma_{xx}$  while the purple line for  $\sigma_{yy}$ . The red points are the fitting points of  $\sigma_{xx}$  in the low frequency regime.

Now let's consider  $\sigma_{xx}$ . The behavior of the component  $\sigma_{xx}$  is much different from that of  $\sigma_{yy}$ , which can be observed from Figure 6 when the frequency is very small. The logarithmic behavior of  $\sigma_{xx}$  reminds us of the Drude model which is a classical description of the electrical conductivity in a metal:

$$\text{Re}(\sigma)_{\text{Drude}} = \frac{\sigma_0}{1 + \omega^2 \tau^2}, \quad (39)$$

where,  $\sigma_0 = ne^2\tau/m$  is the DC conductivity of  $\sigma_{xx}$ ,  $n, e, m, \tau$  are respectively the electron's number density, charge, mass and the mean free time between the ionic collision. For  $T/\sqrt[3]{\rho} \approx 0.04$ , we fit the  $\tau$  and  $\sigma_0$  for various  $\alpha$  in low frequencies. In Figure 6, the blue lines represent the logarithmic behavior of  $\text{Re}(\sigma_{xx})$  while the purple lines stand for the logarithmic behavior of  $\text{Re}\sigma_{yy}$ . The red points in Figure 6 are the fitting points for  $\text{Re}\sigma_{xx}$

in low frequencies. The values of  $\tau$  and  $\sigma_0$  can be read from the fitting points as

$$\begin{aligned}
\sigma_{0(\alpha=-0.19)} &\approx 1.09082 * 10^{10}T, & \tau_{(\alpha=-0.19)} &\approx 1.34592 * 10^8 T^{-1}, \\
\sigma_{0(\alpha=-0.1)} &\approx 6.07532 * 10^9 T, & \tau_{(\alpha=-0.1)} &\approx 8.55262 * 10^7 T^{-1}, \\
\sigma_{0(\alpha=0.0001)} &\approx 3.26748 * 10^9 T, & \tau_{(\alpha=0.0001)} &\approx 5.47106 * 10^7 T^{-1}, \\
\sigma_{0(\alpha=0.1)} &\approx 2.18399 * 10^8 T, & \tau_{(\alpha=0.1)} &\approx 2.72375 * 10^7 T^{-1}, \\
\sigma_{0(\alpha=0.2)} &\approx 6.11933 * 10^7 T, & \tau_{(\alpha=0.2)} &\approx 1.13346 * 10^7 T^{-1}, \\
\sigma_{0(\alpha=0.25)} &\approx 4.22257 * 10^7 T, & \tau_{(\alpha=0.25)} &\approx 7.43723 * 10^6 T^{-1}.
\end{aligned} \tag{40}$$

We see that in general, both the DC conductivity and the mean free time  $\tau$  decrease as  $\alpha$  grows.

#### IV. CONCLUSIONS

In this paper we studied the holographic p-wave superconductors in a five-dimensional Einstein-Gauss-Bonnet gravity theory with an SU(2) Yang-Mills gauge field. We treated the SU(2) Yang-Mills field as a probe field, which means the back reaction of the Yang-Mills field on the background is not taken into account. A component of the vector field will condense when the temperature of the Gauss-Bonnet black hole is below a critical value. This condensation is interpreted as a p-wave superconducting phase transition on the boundary field theory. We found that when the Gauss-Bonnet coupling increases, the value of the condensation becomes bigger and the critical temperature decreases, as in the case of holographic s-wave superconductors. This means a positive Gauss-Bonnet term makes the condensation harder. This phenomena is also observed from the gap frequency  $\omega_g$  which increases as the Gauss-Bonnet coupling grows.

The electric conductivity of the p-wave superconductor is quite different from the one for s-wave superconductors. The conductivity perpendicular to the direction of the condensation behaves like a s-wave one, while the conductivity parallel to the direction of the condensation behaves much different. In the low frequency regime, this conductivity can be well explained by the Drude model. As for the effect of the Gauss-Bonnet term, both the DC conductivity and the mean free time decrease as  $\alpha$  increases. In the low frequency regime, we obtained the mass of excited quasi-particles  $\Delta$  for  $\sigma_{yy}$ . We found that in this holographic model, the usual relation  $\omega_g = 2\Delta$  in BCS theory does not hold, which demonstrates the strong coupling between excited quasi-particles. For the conductivity  $\sigma_{xx}$ , fitting the data, we obtained the DC conductivity and mean free time in the low frequency regime.

In particular, we observed that the condensation for the case of  $\alpha = 0.25$  and the relation between the gap frequency and the mass of quasi-particles for the cases of  $\alpha = 0.20$  and  $0.25$  behaves strange. This strange behavior is consistent with the causality bound on the Gauss-Bonnet coefficient from the dual field theory [36, 37]. The latter imposes a constrain on the coefficient:  $-7/36 \leq \alpha \leq 9/100$ . This implies that our results for the cases of  $\alpha > 9/100$  are not trustable.

Note that here the superconducting phase transition is still second order. Namely the Gauss-Bonnet term does not change the order of the phase transition and some critical

exponents. They still take the mean-field theory values. On the other hand, the back reaction of the  $SU(2)$  Yang-Mills field will change the phase transition from second order to first order when the ratio of the gravitation constant to the Yang-Mills coupling reaches a critical value [17]. It would be interesting to see how the Gauss-Bonnet term affects the phase transition when the back reaction is included.

### Acknowledgements

RGC thanks the organizers and participants for various discussions during the workshop on Dark Energy and Fundamental Theory held at Xidi, Anhui, China, May 28-June 6, 2010, supported by the Special Fund for Theoretical Physics from the National Natural Science Foundation of China with grant No: 10947203. ZYN and HQZ would like to thank M.M. Roberts and B. Hu for helpful discussions. ZYN would like to thank B.N. Lu and Y.W. Sun for useful discussions. This work was partially supported by the National Natural Science Foundation of China (No. 10821504 and No. 10975168).

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